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Interface cohesive elements to model matrix crack evolution in composite laminates

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Abstract:	In this paper, the transverse matrix (resin) cracking developed in multidirectional composite laminates loaded in tension was numerically investigated by a finite element (FE) model implemented in the commercially available software Abaqus/Explicit 6.10. A theoretical solution using the equivalent constraint model (ECM) of the damaged laminate developed by Soutis et al. was employed to describe matrix cracking evolution and compared to the proposed numerical approach. In the numerical model, interface cohesive elements were inserted between neighbouring finite elements that run parallel to fibre orientation in each lamina to simulate matrix cracking with the assumption of equally spaced cracks (based on experimental measurements and observations). The stress based traction-separation law was introduced to simulate initiation of matrix cracking and propagation under mixed-mode loading. The numerically predicted crack density was found to depend on the mesh size of the model and the material fracture parameters defined for the cohesive elements. Numerical predictions of matrix crack density as a function of applied stress are in a good agreement to experimentally measured and theoretically (ECM) obtained values, but some further refinement will be required in near future work.
Response to Reviewers:	Reviewers comments on paper ACMA-D-13-00409: Interface cohesive elements to model matrix crack evolution in composite laminates Y. Shi, C.Pinna and C. Soutis* Specific comments: 1. “In abstract and elsewhere: the sentence "assumption of equally spaced cracks (based on experimental measurements…)…" should be revised. In fact at low crack density the crack location is random and only at high crack density , close to the "characteristic damage state" introduced by Reifsnieder the crack distribution becomes more uniform.” Answer: We agree, and But in this work the matrix cracking was attempted to model in a macro-scale model. this is why it is mentioned as an assumption for the macro-scale FE model. In order to simulate the random location of matrix cracking generated, a micro-scale FE model or other method such as Discrete Element Method (DEM) will be

required which is not attempted in this paper. The current results for numerical prediction of crack initiation and growth were reliable because the crack density was always numerically predicted in the same stress range compared to experimental data, even though the different meshing size was performed. Therefore, this numerical method can be accepted as an effective way to predict matrix cracking with the assumption of “equally spaced cracks".

2. “Why there are so many references to papers with impact loading? Kind of misleading regarding the subject. May be instead more papers with different approaches to cracking evolution should be referred?”

Answer: In fact there are listed several papers on matrix cracking prediction, see Ref 13-14, 25-26,28 and Ref. 35-42. Papers on impact are included because of the previous publication by the authors that focused on the prediction of impact induced damage, and some related material properties used in the present paper, appeared in that publication. In addition, the cohesive elements presented in the present study to predict matrix cracking were used in the impact work to simulate delamination (interlaminar cracking rather than intralaminar).

3. “Is it really + sign in eq (2)?”

Answer: Yes, it is confirmed by the original publications on ECM.

4. “Eq (5) : definition h1 and h2 for ply thicknesses are not given. Still not clear if h2 is the whole 90-thickness or ½ of it. From the form of (5) and (6) and (10) seems to be 1/" “

Answer: In this work, the ply thickness is 0.132mm. The parameters h1 and h2 are defined in the manuscript and represent the thickness of the off-axis plies and 90o plies, respectively

5. “Before eq (11): the R-curve concept is very old and comes from individual crack in metals when it becomes larger. In transverse cracking case all cracks (even at the high stress) are of the same size. Therefore, the meaning of the R- curve should be discussed/explained. Could it be reflecting the effect of statistical distribution of fracture initiation/propagation properties in the specimen? “

Answer: True, the R-curve concept comes from the fracture of metals where a single crack develops. This has been used extensively in the composites literature and represents the resistance to grow multiple cracks within a ply. The mathematical expression of Eq.11 simply describes initiation and growth of transverse cracking and is expressed in terms of crack density D rather than crack length, which is explained in the manuscript.

6. “In (12) Go and R are fitting parameters. It is clearly stated and the values are shown in Fig. 5 and 6. What is difficult to accept, is that the values of parameters for the same material are different if the cracked ply thickness change. This limits the application of the approach significantly. Predictions are possible only for the given material with the same ply thickness but in different laminate lay-ups”

Answer: The fact that the fracture parameters for initiation and growth vary with lay-up comes from experimental measurements and observations. The analytical model simply is trying to capture the observations. The authors agree that the fracture toughness should be material property but then composite laminates are not homogeneous materials but rather structures and the stacking sequence does have an effect on initiation and propagation. In the ECM model if the parameters remain unchanged the stress for initiation and maximum crack density will be underestimated, which of course will lead to a more conservative design, no harm there. This is better explained in the revised manuscript.

7. “The description of the "numerical damage model" is not sufficiently clear. Definitions are missing or "diffuse". Examples:

After (14) "… the material stiffness" is actually the cohesive element stiffness

Before (17): "… criterion [31] can be used…". How do you know?

After (17) : what is "beta"

In (17) : is there also the R-curve for Gic used? If so, Gr should be written instead of Gic. It should be told that Giic is not needed in the current paper”

Answer: text has been amended, and it is the cohesive element stiffness.

Before (17): "… criterion [31] can be used…". Of course, other fracture criteria could be used to simulate matrix crack formation, but in this study this BK law has been selected and it appears that can successfully capture experimental observations.

After (17) : what is "beta": Parameter β is the mode mixity ratio and is defined in the revised manuscript.

In (17) : is there also the R-curve for Gic used? If so, Gr should be written instead of Gic. It should be told that Giic is not needed in the current paper”

Answer: In this section, the numerical model was introduced that employs cohesive elements to simulate the matrix cracking (or delamination in the previous impact paper by the authors). Parameters GIC and GIIC denote the fracture toughness of the composite system used for fracture modes I and II, respectively. The authors agree that mode I may be the dominant one for the loading case examined but the FE model to run requires both values to be defined. The FE model does not need the G0 and R parameters used in the ECM approach as fitting parameters.
8. “Finite element model" gives more questions than answers:
"what is depth of each individual ply"? Is it the size y-direction or z-direction? The size 0.132 is like a thickness of a ply. Only one element in thickness direction? Details about the number of elements/nodes has to be given.
Answer: The depth of each individual ply is 0.132 mm along the z direction, axes are defined in the revised manuscript and a typical FE mesh is provided in the new figure 5.
Is it a 3-D analysis as stated in the first sentence or 2D? There is nothing about edge effects (possible initiation at edges and propagation along fibers). Therefore I conclude that the analysis was 2D.
Answer: It is a 3D model. A figure to illustrate the 3D model with dimensions and boundary conditions has been added in the manuscript, see Fig. 5.
Was the whole specimen modeled or repeating elements of certain length (density) considered
Answer: The size of the model used is 10mm x 10mm to represent the area of cracks generated based on a certain crack density which is needed to simplify the model and reduce the computing time. It could be viewed as an RVE approach that uses repeating elements of certain length.
How about the effective constraint? Was it used or each layer was modeled separately? If so, boundary conditions have to be described that give "repeating element"
Answer: In this FE model, a displacement was applied at both ends of the plate, as shown in Fig. 5. The applied displacement is calculated based on the material properties and the stress value measured by the experiment. The corresponding description was added in the first paragraph of section 3.2 in the manuscript.
"all the 90-plies were located in the middle plane of the laminate" is an incorrect expression
Answer: The manuscript has been changed.
"the stiffness will be gradually degraded" is the stiffness of the cohesive element not the material
Answer: Text has been corrected.
"and a crack density of 2 cracks/mm was assumed…… which corresponds …. to 20 cracks per cm" is really a very deep and correct explanation. Should it be given?”
Answer: Text has been modified
9. “Results and discussion and conclusions
a. "the fracture model was found to depend on this ratio, so the same fracture parameters were used for both lay-ups" What does it mean?
Answer: Based on the experimental measurement, the GIC, G0 and R will influence the predicted accuracy using ECM for different thickness ratios. For the prediction of [0/90]s and [25/-25/902]s the stacking sequence and thickness of laminates are different but the thickness ratio is same (=1). So the same parameters of GIC, G0 and R were used for ECM prediction of these two lay-ups, see also previous comments.
b. "mesh refinement can slightly improve the accuracy…". This is NOT what we see in Fig. 7. We see that refinement is REDUCING THE AGREEMENT with test data at low crack density,
Answer: For the [0/90]s lay-up, the initial crack was found at a little higher stress value when the refined model was used but the initial crack density value was reduced to 1 crack/cm which is well matched with experimental data than that predicted (2 cracks/cm) by the relatively coarse model. Moreover, it did also improve the crack density for the [25/-25/902]s. But, improvements are relatively small and this is why the coarser mesh is recommended that speeds up the solution and results are acceptable taking into account the experimental uncertainties in measuring fracture parameters or accurately measuring crack densities.
c. It is pointless to discuss 0.5mm or smaller distance between cohesive elements. The distance is scaled with the size of the crack (90-layer thickness) and it has to be discussed in these terms. By the way, the ply thickness should be given in Table 1. “
Answer: For the FE method, due to the initial size of model (distance defined between

	cohesive elements) was determined based on the experimental observation, it needs to be investigated the mesh size effect for the prediction for a different crack density was defined at a saturation level. In addition, the mesh dependency of cohesive elements were unknown for this simulation, it is more important to perform a refined model with refined size of the whole model (including cohesive elements). The results showed the refinement did not give much improvement but the crack density was accurately predicted in the same stress range when compared to the experimental data; this gives confidence to the proposed FE method, which is a reliable way to predict crack density and identify parameters that have an effect when simulating fracture of complex laminated structures. It is also a way of validating failure criteria, stress and/or fracture based. The ply thickness has been added in Table 1.

Interface cohesive elements to model matrix crack evolution in composite laminates

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Abstract

In this paper, the transverse matrix (resin) cracking developed in multidirectional composite laminates loaded in tension was numerically investigated by a finite element (FE) model implemented in the commercially available software Abaqus/Explicit 6.10. A theoretical solution using the equivalent constraint model (ECM) of the damaged laminate developed by Soutis et al. was employed to describe matrix cracking evolution and compared to the proposed numerical approach. In the numerical model, interface cohesive elements were inserted between neighbouring finite elements that run parallel to fibre orientation in each lamina to simulate matrix cracking with the assumption of equally spaced cracks (based on experimental measurements and observations). The stress based traction-separation law was introduced to simulate initiation of matrix cracking and propagation under mixed-mode loading. The numerically predicted crack density was found to depend on the mesh size of the model and the material fracture parameters defined for the cohesive elements. Numerical predictions of matrix crack density as a function of applied stress are in a good agreement to experimentally measured and theoretically (ECM) obtained values, but some further refinement will be required in near future work.

Keywords: Composite laminates; Finite element analysis; Cohesive elements; Crack density; Equivalent constraint model; Damage; Matrix cracking

1. Introduction

Advanced composite materials offer high specific strength and stiffness properties and have been widely used in the aerospace industry, especially for the fabrication of structural components in military and more recently civil aircraft. Fibre reinforced plastics, such as thermosets or thermoplastics reinforced with carbon or glass fibres have taken the

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place of the conventional metal alloys in the aerospace industry [1]. However, composite laminates when subjected to quasi-static or tensile fatigue loading exhibit relatively brittle behaviour and poor damage resistance, especially those earlier composite systems with untoughened thermoset resins. This can be a critical design issue and limitation for structural applications requiring high damage resistance [2-3]. Failure of composite laminates is a complicated process including intra- and inter-laminar (delamination) damage, which leads to stiffness loss and load-carrying capability as the damage becomes more extensive [4]. In general, intra-laminar damage occurs within a single lamina in the form of tensile and/or compressive matrix damage, debonding between the fibre and resin interface and at higher applied loads tensile and/or compressive fibre breakage that leads to final failure of the laminated construction [5-7]. Matrix cracking (or intra-laminar cracking) and axial splitting along the fibre direction have been recognised as early damage mechanisms in transverse loading due to resin-dominated behaviour. Much attention has been paid to these damage modes due to the resulting stress concentration at the crack tip; this may induce delamination as inter-laminar damage that occurs between neighbouring plies, which may lead in fibre breakage and complete loss of load-carrying capability [8-12]. These modes of damage highlight the importance of investigating and understanding their initiation and evolution in composite laminates with the aim to select lay-up configurations that show better damage resistance and tolerance.

A large number of theories have been published to predict matrix cracking based on stress-based failure criteria or damage/fracture mechanics. Polynomial failure criteria, such as the Tsai-Wu or Tsai-Hill, are based on the equivalent stress or strain. They are usually employed to describe the failure envelope of any given multidirectional laminate subjected to multiaxial loading. However, the damage mechanisms of different modes cannot be clearly identified using such failure criteria. Hashin developed an effective method to model matrix cracking as a plane problem [13, 14] and it was further developed by Nairn [15], Varna [16] and Berglund [17, 18]. For cross-ply laminates Soutis et al. applied the equivalent constraint model (ECM) to predict the crack density as a function of applied load and stiffness reduction based on a 2D shear-lag analysis [4, 8-12, 19-20]. Continuum damage mechanics (CDM) initiated by the work of Kachanov [21] and Rabotnov [22] is also a popular way of modelling damage in composite laminates

[23-26]. Shi and Soutis [27] attempted to combine different intra- and inter-laminar damage criteria with nonlinear shear behaviour to simulate impact induced damage. The matrix cracking and delamination were accurately captured by a numerical damage model subjected to different impact energy levels. Although methods have been published that predict the extent of the damaged area, there are few methods that simulate the process of matrix cracking within a damaged region.

This paper presents a numerical model that was developed to simulate the growth of transverse matrix cracking by inserting cohesive elements in each lamina between adjacent finite elements along the fibre direction where ultimate crack density (saturation level) is selected based on experimental observations; this helps to define sufficient number of cohesive elements without unnecessarily slow down the numerical solution. Finite element (FE) models were built for composite laminates with various off-axis dominated stacking sequences, $[\pm\theta_m/90_n]_s$. The optimal mesh size for the model was determined by experimentally measured crack density and uniform crack spacing in each ply was assumed, as shown in Fig. 1. The ECM was also used to estimate the crack density for these laminates and analytical and numerical predictions were validated by measurements. The advantage of employing FE is that other damage modes, like delamination and more complex loading conditions, such as multi-axial in plane and out-of-plane loading, can be simulated that is difficult to be achieved by analytical methods, concepts that are not considered in the current analysis.

2. Theoretical model

The equivalent constraint model (ECM) is a theoretical approach used to predict matrix cracking in multidirectional laminates under multiaxial in plane loading and a description of main assumptions and simplifications are discussed here for the reader's benefit. It was assumed that cracks in a damaged lamina are uniformly spaced, which is crucial to solving problems by analysis of a representative volume element. A schematic typical ECM with a damaged lamina is shown in Fig. 2. The layer, k denotes the damaged lamina and all plies above and below the k^{th} ply are replaced with homogeneous layers (I and II), which are governed by the equivalent constraint effect. The stiffness properties of equivalent constraint layers can be obtained by the laminate plate theory (LPT), which provides the stress and strain relationship.

Due to the symmetry of a $[\pm\theta_m/90_n]_s$ laminate, as shown in Fig. 1(for a $[0/90]_s$ lay-up), the analysis was reduced to one quarter of the representative segment. Matrix cracking in the 90^o ply was expected to be the first damage mode to occur. Stresses can be calculated from the stiffness of the constrained homogeneous layers and the modified stiffness of the cracked ply. In order to determine stresses in the damaged ply, it was assumed that the total strain in the individual lamina was equivalent to that in the laminate (implying continuity). This is given by,

$$\overline{\varepsilon}_i^{(k)} = \overline{\varepsilon}_i \qquad k = 1, 2 \tag{1}$$

$\overline{\varepsilon}_i^{(k)}$ and $\overline{\varepsilon}_i$ denote the total strain vectors of the k^{th} layer and laminate, respectively. Thus, the average constitutive equations of a damaged lamina can be expressed:

$$\overline{\sigma}_i^{(k)} = Q_{ij}^k \left(\overline{\varepsilon}_j + \overline{\varepsilon}_j^{0(k)}\right) \qquad k = 1, 2 \tag{2}$$

where $\overline{\sigma}_i^{(k)}$ represent the total stress vector of the constraint layers (k =1) and the damaged 90^o ply (k =2), respectively. $\overline{\varepsilon}_j^{0(k)}$ is the residual thermal strain vector of the k^{th} layer. Q_{ij}^k is the stiffness of the constraining layers (k =1) and modified reduced stiffness of the damaged 90^o ply (k =2). The reduced stiffness matrix of the damaged ply can be derived by the in-situ damage effective function (IDEF), Λ_{ij}, as a function of crack density (a 2D shear lag stress analysis is followed) [19].

Then the laminate stress can be written using the classical laminate plate theory:

$$\overline{\sigma}_i = \frac{1}{(1+\chi)}(\overline{\sigma}_i^{(2)} + \chi\overline{\sigma}_i^{(1)}) \tag{3}$$

where χ is the thickness ratio of the constraining layer over the thickness of the 90^o layer. The constitutive relation of the cracked laminate is obtained by combining Eqs 2 & 3

$$\overline{\sigma}_i = \overline{Q}_{ij}(\overline{\varepsilon}_j - \overline{\varepsilon}_j^{p}) \tag{4}$$

where $\overline{Q}_{ij}$ is the in-plane stiffness matrix of the damaged laminate. The $\overline{\varepsilon}_j^{\,p}$ is a permanent strain, which represents the effect of interaction of damage and residual stresses and is defined as:

$$\overline{\varepsilon}_j^{\,p} = -\overline{S}_{ij} \frac{1}{(h_1 + h_2)} \sum_{k=1}^{2} h_k Q_{jl}^k S_{lm}^{0(k)} \overline{\sigma}_m^{0(k)} \tag{5}$$

where $\overline{S}_{ij}$ is the in-plane compliance matrix.

Consider a $[\pm\theta_m/90_n]_s$ laminate with a finite gauge length of $2l$ and width of w with transverse ply cracks. The potential energy is written as:

$$PE = U - 2(h_1 + h_2)w2l\overline{\sigma}_i\,\overline{\varepsilon}_i \tag{6}$$

where U is the total strain energy of the laminate. Using the constitutive relation defined in Eq. (2), the total strain energy is found

$$U = \frac{1}{2}\sum_{k=1}^{2} 2lw2h_k Q_{ij}^k (\overline{\varepsilon}_i + \overline{\varepsilon}_i^{0(k)})(\overline{\varepsilon}_j + \overline{\varepsilon}_j^{0(k)}) \tag{7}$$

The energy release rate is defined as the first partial derivative of the potential energy corresponding to the crack surface area, A, with a fixed applied laminate stresses

$$G = -\left(\frac{\partial PE}{\partial A}\right)\bigg|_{\overline{\sigma}_i} \tag{8}$$

Rearranging Eqs 4-7 and substituting them into Eq. 8 gives the energy release rate associated with matrix cracking, which can be derived and expressed as [20]:

$$G\left(\sigma, D^a\right) = -h_2 \frac{\partial Q_{ij}^{(2)}}{\partial D^a}\left[\overline{S}_{il}\,\overline{S}_{jm}\,\overline{S}_l\,\overline{S}_m + 2\overline{S}_{il}\,\overline{\sigma}_l\left(\overline{\varepsilon}_j^{0(2)} + \overline{\varepsilon}_j^{\,p}\right) + \left(\overline{\varepsilon}_i^{0(2)} + \overline{\varepsilon}_i^{\,p}\right)\left(\overline{\varepsilon}_j^{0(2)} + \overline{\varepsilon}_j^{\,p}\right)\right] \tag{9}$$

where D^a denotes the total crack surface area per unit length and width of laminate:

$$D^a = 2h_2 C_d \tag{10}$$

In equation (10) C_d is the average crack density. The second and third terms inside the bracket on the right hand side in Eq. 9 represent the effect of the residual stresses and interaction with damage. The energy release rate shown in Eq. 9 is easily derived if the in-plane stiffness matrix of the cracked laminate is known for a given crack density.

In general, the resistance of the composite to transverse matrix cracking increases with crack density when loaded under quai-static uniaxial tension. Thus, a resistance curve, analogous to the R-curve concept in fracture mechanics can be used to predict the resistance to propagation of transverse cracking [28-30].

$$G(\sigma, D^a) = G_R(D^a) \tag{11}$$

where G_R is the laminate resistance to multiple transverse ply cracking. The R-curve was previously found to be dependent on the 90^o ply thickness but independent of the stiffness of the constraining layers by investigating the relation between G_R and crack density C_d for different composite laminates. A simple mathematical expression for G_R was derived by curve fitting [30]:

$$G_R = G_{IC} + G_0(1 - e^{-RD}) \tag{12}$$

where D is the crack density function, G_{IC} is the critical energy release rate associated with mode I matrix cracking, while G_0 and R are considered as material/laminate constants that capture the resistance to crack growth with increasing applied load/stress.

3. Numerical damage model

The commercial FE software package Abaqus/Explicit 6.10 was employed to predict transverse matrix cracking as a function of applied tensile stress by running a numerical program with cohesive elements. The traction-separation law was used to predict the growth of the matrix cracking under mixed-mode loading, section 3.1. An appropriate FE model was built with certain kinematic and loading boundary conditions that are discussed in section 3.2.

3.1 Cohesive elements

In recent work on impact induced damage by the authors [27], interlaminar cracking (delamination) was successfully modelled by numerical methods using cohesive elements. A quadratic stress failure criterion was employed to predict delamination initiation. Delamination propagation based on fracture mechanics was proposed by Camanho and Dávila [31] where cohesive elements were introduced at each interface of neighbouring plies in the composite model. The stress failure criterion used to estimate the onset of damage is given by:

$$\left(\frac{\sigma_n}{N}\right)^2+\left(\frac{\sigma_s}{S}\right)^2+\left(\frac{\sigma_t}{T}\right)^2=1 \tag{13}$$

where σ_i $(i = n, s, t)$ denotes the traction stress vector in the normal n and shear directions, s and t, respectively, while N, S and T are defined as the corresponding inter-laminar normal and two shear strengths, Fig.3a.

The traction stress σ_i can be calculated as given in the Abaqus manual [32] using the stiffness in Modes I, II and III and the opening and/or sliding displacements δ_i:

$$\sigma_i = K_i\delta_i, \qquad i = n, s, t \tag{14}$$

Once damage (in the form of a crack) has initiated, the stiffness of the cohesive element is gradually degraded in terms of a damage variable d ranged from zero, when damage initiates, to one when the interface element is completely damaged. Mixed-mode loading in terms of the energy release rates associated with Modes I, II and III is used to predict damage growth. For a linear softening process the damage variable d for evolution is defined as:

$$d=\frac{\delta_m^f\left(\delta_m^{\max}-\delta_m^0\right)}{\delta_m^{\max}\left(\delta_m^f-\delta_m^0\right)} \tag{15}$$

where δ_m^{max} refers to the maximum value of the mixed-mode displacement attained during the loading history. The δ_m parameter corresponds to the total mixed-mode displacement (normal, sliding, tearing) given by:

$$\delta_m=\sqrt{\delta_n^2+\delta_{shear}^2}=\sqrt{\delta_n^2+\delta_s^2+\delta_t^2} \tag{16}$$

In Eq. (15) δ_m^f is the mixed-mode displacement at complete failure and δ_m^0 is the effective displacement at damage initiation. A Benzeggagh–Kenane (B-K) fracture energy based criterion [31] can be used to define the mixed-mode displacement for complete failure, δ_m^f:

$$\delta_m^f = \begin{cases} \frac{2}{k\delta_m^0}\left[G_{IC} + \left(G_{IIC} - G_{IC}\right)\xi^{\eta}\right] & \delta_n > 0 \\ \sqrt{\left(\delta_s^f\right)^2 + \left(\delta_t^f\right)^2} & \delta_n \leq 0 \end{cases} \tag{17}$$

where η is the B-K power law parameter that can be determined using a least-square fit from a set of mixed-mode bending experimental data; $\xi = \frac{\beta^2}{1+\beta^2}$ with ξ taking values between zero and one. When ξ=0 the crack is mode I driven, while as $\xi \to 1$ fracture is mode II dominated (and this is also the case when exponent η=0). Parameter β above is the mode mixity ratio.

A typical linear traction-separation model used for fracture Modes I, II and III is shown in Fig. 3b. Initially, the linear elastic response is represented using the stiffness terms K_i *(i = n, s, t)* until the normal and shear strengths are reached. Beyond these strength values, the stiffness will start to be linearly reduced according to the damage evolution variable *d* defined in Eq. (15) and finally complete damage occurs when the maximum displacement is reached. This damage modelling approach is implemented in the FE model described in section 3.2, but in this analysis the crack is within the transverse ply (intra-) rather than between plies (inter-laminar cracking or delamination).

3.2 Finite element model

A 3D FE model representing unidirectional tensile loading was built from eight-node linear brick elements, C3D8R. The depth of each individual ply was represented in the model by one element with a thickness of 0.132 mm. Cohesive elements, COH3D8, of zero-thickness were inserted between neighbouring finite elements parallel to the fibre orientation within the 90^o ply(s), Fig. 4. Modelling parameters were determined by consideration of the convergence of computing and the accuracy of numerical prediction (when compared to measurements); a model that is too refined uses an excessively large number of solid and cohesive elements. Conversely, too small a model would underestimate stresses and introduce errors in the numerical prediction of the matrix crack density. Composite laminates with the stacking sequences of $[0/90_n]_s$ and $[\pm25/90_j]_s$ were investigated in this study, for which all the 90^o plies were inner plies . Local coordinates were created to help define the orientation of each lamina and to build the layers of cohesive elements in each ply. As an example, a 3D FE model for a $[0/90]_s$ lay-up is shown in Fig.5, where an axial displacement is applied at both ends of the plate; the full model consists of 2205 nodes and 2360 elements (that includes both brick and cohesive elements), and was solved in approximately 30 minutes.

A mixed-mode traction-separation law was used to define the evolution of transverse matrix cracking as discussed in section 3.1. If the damage criteria are not satisfied, separation (matrix cracking) does not occur and adjacent elements will be perfectly connected. Otherwise matrix cracking will initiate and the stiffness of the cohesive elements will be gradually degraded following the linear softening law described by equation (15).

The mesh size of the model is a crucial issue that dictates the numerical efficiency and accuracy with which the transverse matrix crack formation and crack density can be modelled; here, the selected mesh density was based on experimental measurements and a crack density of 2 cracks/mm was assumed that could be reached at saturation level. The present analysis neglects cracks that could develop in other ±θ off-axis plies or local delamination, which could influence the predicted results.

The composite laminates examined are made of a 934 epoxy resin reinforced with unidirectional T300 carbon fibres. Detailed material properties are listed in Table 1 [20]. The properties of the cohesive elements are also presented in Table 2 and include the elastic stiffness, strength and fracture energies [33-35]. The accuracy of the analysis strongly depends on the stiffness of the interface element [36]. High stiffness can prevent interpenetration of crack faces but might lead to numerical problems. Daudeville et al. [37] proposed normalisation of the interface stiffness in terms of a small thickness t (10^{-2} mm) in the resin rich zone of the composite laminate from which a high relative stiffness can be obtained. Several authors have proposed different values for the interface stiffness and some of these were selected equal to 10^7 N/mm^3 [38], 5.7×10^7 N/mm^3 [39] and 10^8 N/mm^3 [40]. Zou *et al.* [41] proposed a value for the stiffness between 10^4 and 10^7 times the value of the strength of the interface per unit length. In the current work the interface stiffness was taken as 10^6 N/mm^3 for the matrix crack mode, which has been shown [42] to give reasonable predictions for carbon/epoxy laminates. Damage evolution under mixed-mode loading was predicted by the Benzeggagh–Kenane fracture energy law [31], in which a factor of η=1.45 based on experimental data. This however may vary and need to be evaluated for a different composite material system.

4. Results and discussion

In this section, experimental results are compared to numerically predicted crack density as a function of applied stress to assess the validity of modelling transverse matrix cracking by using the cohesive elements; theoretical predictions by the ECM are also presented for comparison purposes.

In Fig. 6 the FE predicted transverse matrix crack density is plotted against applied stress for $[0/90]_s$ and $[25/-25/90_2]_s$ laminates and compared to experimentally measured, and theoretically calculated results. The fracture parameters used in eq.(12) for the theoretical predictions were taken equal to:

$$G_{IC} = 190\ (Jm^{-2}) \qquad G_0 = 125\ (Jm^{-2}) \qquad R = 6.5$$

For the $[0/90]_s$ lay-up the ECM gave an acceptable agreement with experimental data. It can be observed in figure 6 that the crack density rapidly increased after its initiation at an applied stress of around 550 MPa. Propagation slowed after the crack density rose above 10 cracks cm^{-1}. A theoretical maximum crack density of 16 cracks cm^{-1} was obtained at approximately 844 MPa, whereas a maximum crack density value of 15.3 cracks cm^{-1} was experimentally measured. The numerical predictions are also in a good agreement with experiment, especially at high crack densities; a maximum crack density of 16 cracks cm^{-1} was found at an applied stress of 830.1 MPa. FE and ECM results also compared favourably with experimental data for the $[25/-25/90_2]_s$ laminate, Fig.6. The maximum crack density was 6 cracks cm^{-1} predicted by the numerical model, compared to 5.3 cracks cm^{-1} obtained experimentally. The two laminates analysed in Fig. 6 have the same thickness ratio χ (=1). The fracture model was found to depend on this ratio, so the same fracture parameters were used for both lay-ups.

In Fig.7 results are presented for the $[0/90_2]_s$ and $[25/\text{-}25/90_4]_s$ laminates, where χ =1/2. The numerical model gave a good prediciton for the $[0/90_2]_s$ lay-up, but underestimated the maximum crack density with a value of 9 cracks cm^{-1}, compared to a measured value of 10.13 cracks cm^{-1}. The FE model also accurately predicted the crack density for the $[25/\text{-}25/90_4]_s$ lay-up giving a maximum crack density of 4 cracks cm^{-1} that is closely to the experimentally measured 4.27 cracks cm^{-1}. The ECM model underpredicted slightly the stress for crack initiation for both laminates, but crack growth is accuratelly captured . It should be noted though that the fracture parameters employed in Eq.(12) were altered to fit better the data presented in Fig.6, i.e.,

$$G_{IC} = 228\ (Jm^{-2}) \qquad G_0 = 178\ (Jm^{-2}) \qquad R = 6.2$$

Soutis et al [19, 20] emphasised that the critical energy release rate G_{IC} and the R-curve values (G_0 and R) differ for various lay-ups, explaining that crack initiation and accumulation are dependent on the thickness ratio χ, which is the thickness of the constraining layers over the thickness of the 90^o plies. It should be said that if the fracture parameters applied for the theoretical prediction of lay-ups with χ =1/2, remain the same as those used for χ =1, then the predicted curve shifts to the left of the experimental data i.e., the stress for crack initiation is underestimated by 14% while the maximum crack density is lower than the measured value by 6.3% for $[0/90_2]_s$ lay-up; the initiation and maximum value of crack density are also underpredicted with a difference of 18.45% and 7.3%, respectively, for the $[25/\text{-}25/90_4]_s$ laminate.

Differences observed between ECM predicted and experimental results may be due to the assumption of uniform crack spacing, definition of fracture parameters and the fact that damage in the constraining plies and local delamination that usually appears at the matrix crack tip were neglected. Finite element modelling using cohesive elements gave reasonable predictions for initiation and accumulation of transverse cracks, especially for the cross-ply laminates. In addition the mesh density used can have an effect on the simulation of transverse matrix cracking. A coarse mesh used in the numerical model indicates an insufficient number of cohesive elements for accurate prediction of the crack density. However, a too refined mesh can prevent successful solution of the program. A cohesive element spacing of 0.5 mm (cohesive elements were inserted 0.5 mm apart) was used to obtain the above results, see schematic of Fig.4. In order to investigate the effect of mesh density, the model was refined with a cohesive element spacing of 0.25 mm and predictions are shown in Fig. 8 together with experimental data for the $[0/90]_s$ and $[25/-25/90_2]_s$ laminates. It can be seen that mesh refinement can slightly improve the accuracy, suggesting the initial mesh was good enough for predicting crack density as a function of applied stress.

Experimental observations have shown that different types of internal transverse cracks existed in the laminates examined, i.e., straight cracks, partial angle cracks and curved cracks in addition to some local delaminations at the crack tip that developed at higher applied loads, nearer to ultimate failure. These damage mechanisms do dissipate energy and delay laminate fracture. Cohesive zone elements could be implemented at the ply interface to simulate delamination, but this is beyond the scope of the current analysis. It should be said though that the FE technique, assuming that the fracture parameters needed are carefully selected, can be used to account for the interaction of different damage modes observed in multidirectional laminates and accurately capture the damage evolution process as a function of applied load(s); further work is required.

5. Concluding remarks

A numerical method using cohesive elements to simulate the transverse matrix cracking was undertaken using the finite element software Abaqus/Explicit 6.10. The equivalent constraint model (ECM) was employed to theoretically predict the matrix crack density with the assumption of uniform crack spacing. The damage parameters used in the theoretical expression of Eq. (12) were obtained by curve fitting of experimental data and assumed constant for cross-ply and off-axis lay-ups with the same thickness ratio χ. In the FE analysis in order to simulate transverse matrix cracking, the cohesive elements were inserted in the interface between neighbouring elements parallel to the fibre direction in each 90^o lamina and the crack spacing was that observed experimentally at saturation level to shorten the computational time. A traction-separation law was applied to predict the initiation and propagation of matrix cracking by appropriately selecting the interfacial stiffness, strength and fracture toughness. A crack spacing of 0.5 mm for positioning the interface elements within the transverse ply was found to give reasonable predictions when compared to crack density measured data. A relatively small improvement was registered for the finer mesh, but this is not recommended since an excessive time was required to build and compute the model. It is thus suggested that a crack spacing of 0.5 mm is good enough, especially when resin cracking in the off-axis plies and local delamination were neglected in the analysis, which can result to further discrepancies. The present work demonstrated that FE with cohesive elements can be used to better understand the effect of certain fracture parameters and failure criteria on crack density evolution and that further work will be required to account for the presence and interaction of more complex damage mechanisms and their impact on stiffness/strength properties and laminate fatigue life.

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Figure captions

Fig. 1 Schematics of a composite laminate with transverse matrix cracking [4].

Fig . 2 A schematic of the Equivalent Constraint Model (ECM) of a damaged laminate (a) Laminate structure (b) ECM model [11].

Fig. 3 Intralaminar cracking represented by cohesive elements (a). Crack modes and coordinates used (b). A schematic of the assumed crack traction-opening or sliding displacement.

Fig. 4 A model of a single transverse ply with interface cohesive elements, inserted at 0.5 mm apart to simulate matrix crack evolution. The circles represent fictitious fibres to simply illustrate their relation to cohesive elements. The ply is modeled as homogeneous orthotropic.

Fig. 5 A typical 3D FE model used for the analysis of a $[0/90]_s$ lay-up.

Fig. 6 Experimental, theoretical and numerical crack densities vs. applied stress for $[0/90]_s$ and $[25/-25/90_2]_s$ laminates.

Fig. 7 Experimental, theoretical and numerical crack densities vs. applied stress for $[0/90_2]_s$ and $[25/-25/90_4]_s$ laminates.

Fig. 8 Crack density vs applied stress for two different cohesive element spacings (mesh size) for $[0/90]_s$ and $[25/-25/90_2]_s$ lay-ups.

Table captions

Table. 1: Material properties for a T300/934 unidirectional laminate [20].

Table. 2: Material parameters for the cohesive elements. [33-35].

colour figure_Revised

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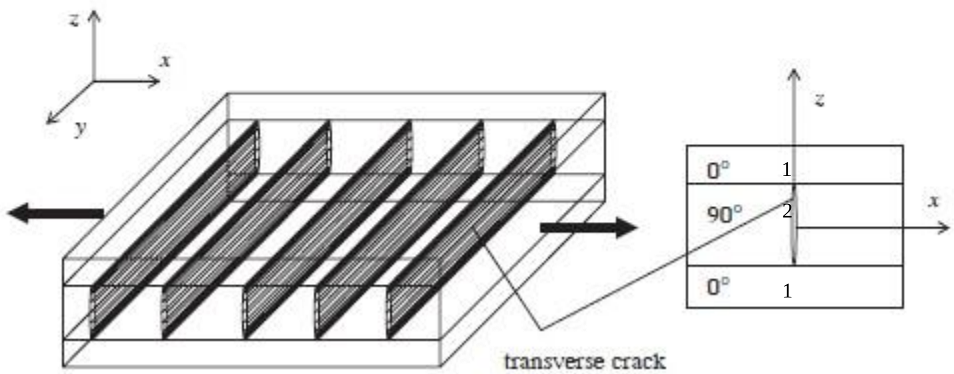
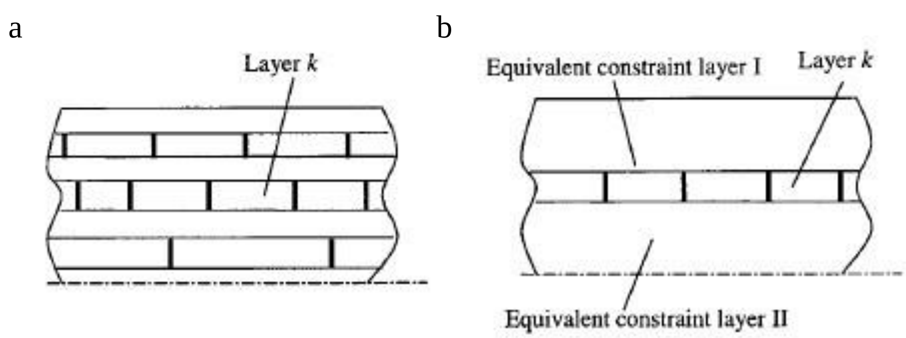


Fig. 1 Schematics of a composite laminate with uniform transverse matrix cracking [4].



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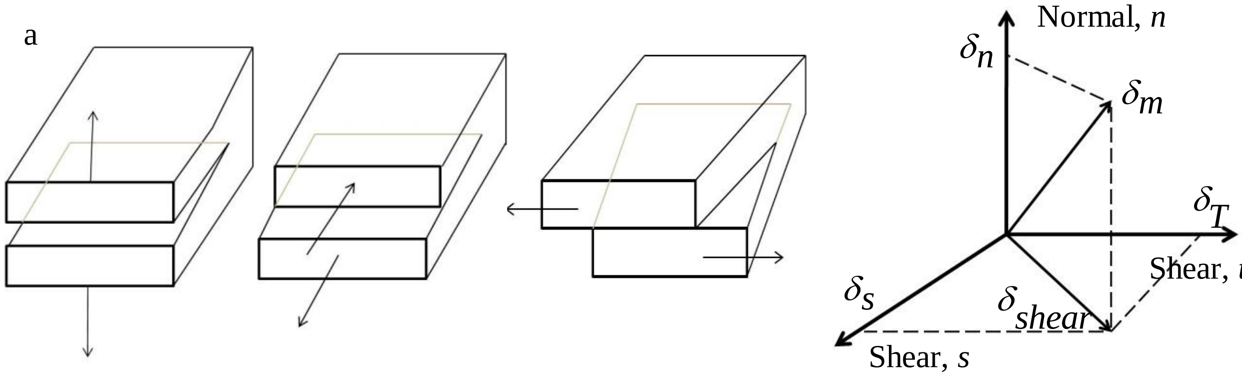
a



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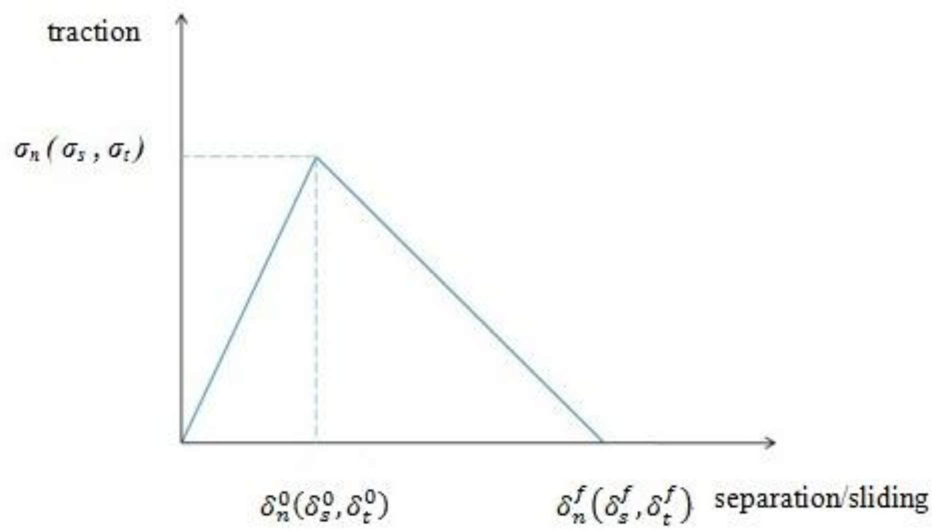

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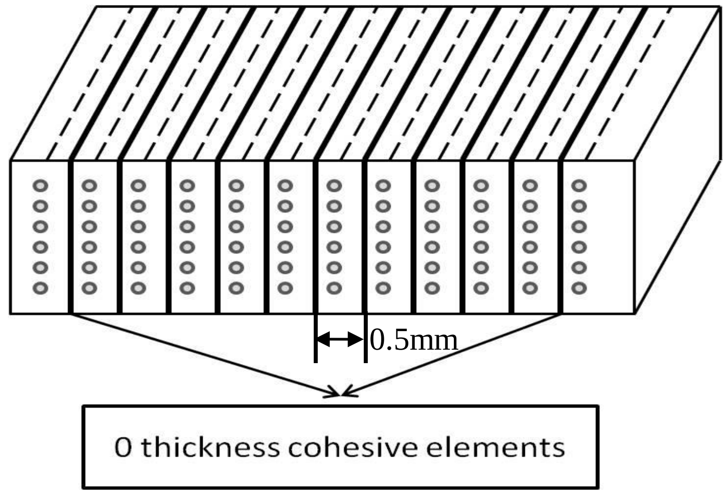

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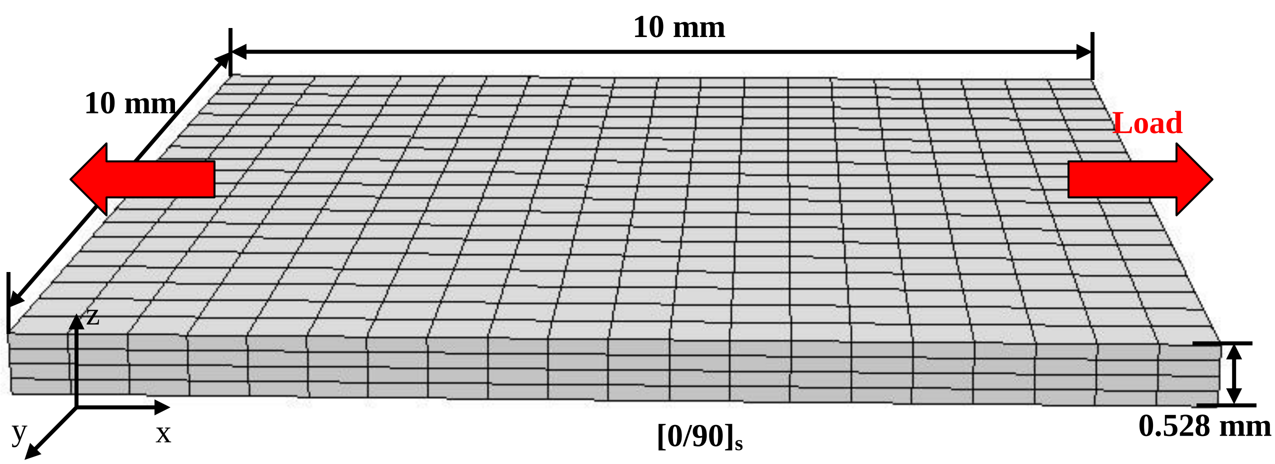

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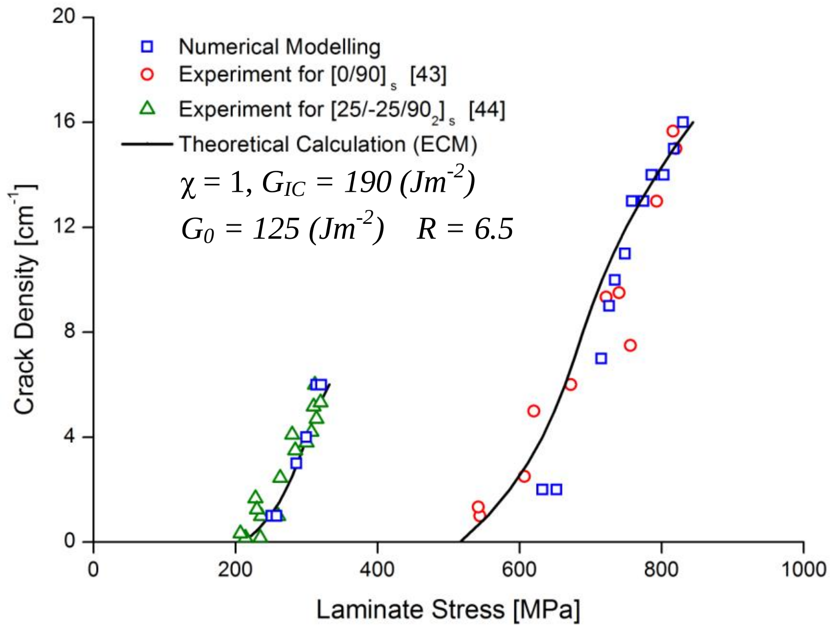
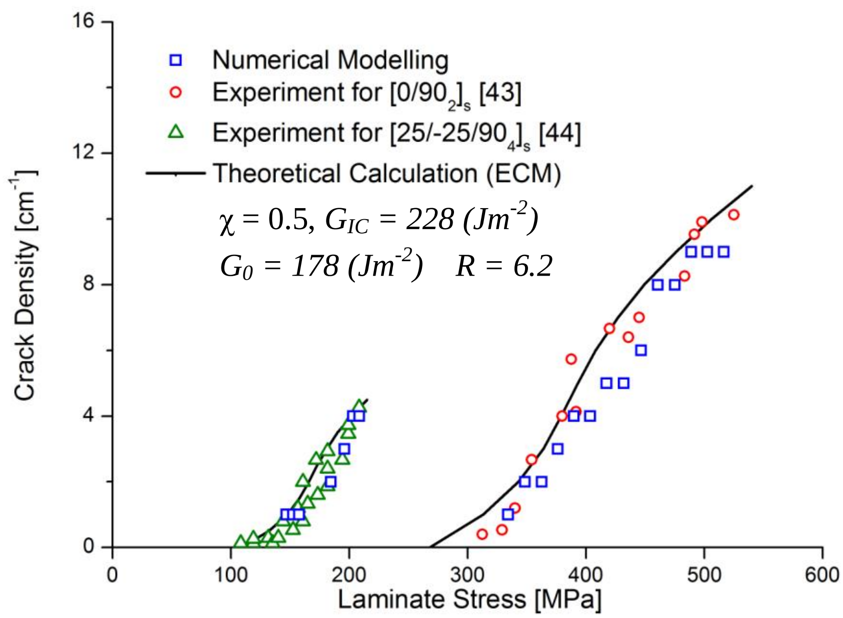


Fig. 6 Experimental, theoretical and numerical crack densities vs. applied stress for $[0/90]_s$ and $[25/\text{-}25/90_2]_s$ laminates.



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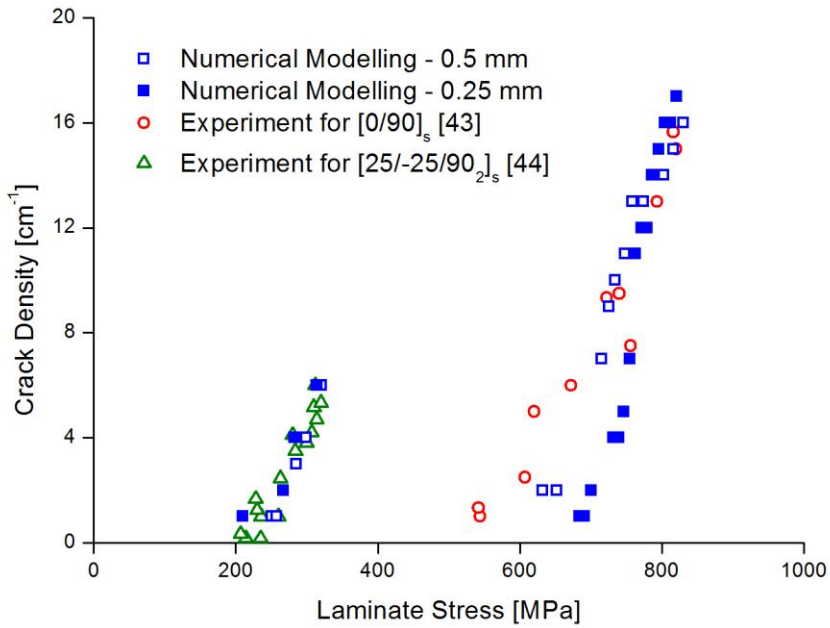

Fig. 8 Crack density vs applied stress for two different cohesive element spacings (mesh size) for $[0/90]_s$ and $[25/-25/90_2]_s$ lay-ups.

Table. 1
Material properties for a T300/934 unidirectional laminate [20]

Longitudinal Modulus (GPa)	144.8	Longitudinal Thermal Expansion Coefficient ($\mu\varepsilon^{o}C^{-1}$)	0.36
Transverse Modulus (GPa)	11.38	Transverse Thermal Expansion Coefficient ($\mu\varepsilon^{o}C^{-1}$)	28.8
In-plane Shear Modulus (GPa)	6.48	Temperature Difference (^{o}C)	-125
Out-of-plane Shear Modulus (GPa)	3.45	Thickness of individual ply (mm)	0.132
Poisson's ration	0.3		

Table. 2
Material parameters for the cohesive elements used in the FE analysis. [33-35]

	Direction, n	Direction, s	Direction, t[1]
Normalised elastic modulus (N/mm^3)	10^6	10^6	10^6
Interface Strength (MPa)	51.7	40	40
Fracture toughness (J/m^2)	190	790	790[2]

Notes: 1) n=normal, s=shear, t=tearing, see Fig.3a

2) This value may differ from that of direction s, but in this study the formation of cracks is mainly affected by modes I and II.

Reviewers comments on paper ACMA-D-13-00409:

Interface cohesive elements to model matrix crack evolution in composite laminates

Y. Shi, C.Pinna and C. Soutis*

Specific comments:

1. "In abstract and elsewhere: the sentence "assumption of equally spaced cracks (based on experimental measurements...)..." should be revised. In fact at low crack density the crack location is random and only at high crack density , close to the "characteristic damage state" introduced by Reifsnieder the crack distribution becomes more uniform."

Answer: We agree, and But in this work the matrix cracking was attempted to model in a macro-scale model. this is why it is mentioned as an assumption for the macro-scale FE model. In order to simulate the random location of matrix cracking generated, a micro-scale FE model or other method such as Discrete Element Method (DEM) will be required which is not attempted in this paper. The current results for numerical prediction of crack initiation and growth were reliable because the crack density was always numerically predicted in the same stress range compared to experimental data, even though the different meshing size was performed. Therefore, this numerical method can be accepted as an effective way to predict matrix cracking with the assumption of "equally spaced cracks".

2. "Why there are so many references to papers with impact loading? Kind of misleading regarding the subject. May be instead more papers with different approaches to cracking evolution should be referred?"

Answer: In fact there are listed several papers on matrix cracking prediction, see Ref 13-14, 25-26,28 and Ref. 35-42. Papers on impact are included because of the previous publication by the authors that focused on the prediction of impact induced damage, and some related material properties used in the present paper, appeared in that publication. In addition, the cohesive elements presented in the present study to predict matrix cracking were used in the impact work to simulate delamination (interlaminar cracking rather than intralaminar).

3. "Is it really + sign in eq (2)?"

Answer: Yes, it is confirmed by the original publications on ECM.

4. "Eq (5) : definition h1 and h2 for ply thicknesses are not given. Still not clear if h2 is the whole 90-thickness or ½ of it. From the form of (5) and (6) and (10) seems to be 1/" "

Answer: In this work, the ply thickness is 0.132mm. The parameters h_1 and h_2 are defined in the manuscript and represent the thickness of the off-axis plies and 90^o plies, respectively

5. "Before eq (11): the R-curve concept is very old and comes from individual crack in metals when it becomes larger. In transverse cracking case all cracks (even at the high stress) are of the same size. Therefore, the meaning of the R- curve should be discussed/explained. Could it be reflecting the effect of statistical distribution of fracture initiation/propagation properties in the specimen? "

Answer: True, the R-curve concept comes from the fracture of metals where a single crack develops. This has been used extensively in the composites literature and represents the resistance to grow multiple cracks within a ply. The mathematical expression of Eq.11 simply describes initiation and growth of transverse cracking and is expressed in terms of crack density D rather than crack length, which is explained in the manuscript.

6. "In (12) Go and R are fitting parameters. It is clearly stated and the values are shown in Fig. 5 and 6. What is difficult to accept, is that the values of parameters for the same material are different if the cracked ply thickness change. This limits the application of the approach significantly. Predictions are possible only for the given material with the same ply thickness but in different laminate lay-ups"

Answer: The fact that the fracture parameters for initiation and growth vary with lay-up comes from experimental measurements and observations. The analytical model simply is trying to capture the observations. The authors agree that the fracture toughness should be material property but then composite laminates are not homogeneous materials but rather structures and the stacking sequence does have an effect on initiation and propagation. In the ECM model if the parameters remain unchanged the stress for initiation and maximum crack density will be underestimated, which of course will lead to a more conservative design, no harm there. This is better explained in the revised manuscript.

7. "The description of the "numerical damage model" is not sufficiently clear. Definitions are missing or "diffuse". Examples:

After (14) "... the material stiffness" is actually the cohesive element stiffness

Before (17): "... criterion [31] can be used...". How do you know?

After (17) : what is "beta"

In (17) : is there also the R-curve for Gic used? If so, Gr should be written instead of Gic. It should be told that Giic is not needed in the current paper"

Answer: text has been amended, and it is the cohesive element stiffness.

Before (17): "... criterion [31] can be used...". Of course, other fracture criteria could be used to simulate matrix crack formation, but in this study this BK law has been selected and it appears that can successfully capture experimental observations.

After (17) : what is "beta": Parameter β is the mode mixity ratio and is defined in the revised manuscript.

In (17) : is there also the R-curve for Gic used? If so, Gr should be written instead of Gic. It should be told that Giic is not needed in the current paper"

Answer: In this section, the numerical model was introduced that employs cohesive elements to simulate the matrix cracking (or delamination in the previous impact paper by the authors). Parameters G_{IC} and G_{IIC} denote the fracture toughness of the composite system used for fracture modes I and II, respectively. The authors agree that mode I may be the dominant one for the loading case examined but the FE model to run requires both values to be defined. The FE model does not need the G_0 and R parameters used in the ECM approach as fitting parameters.

8. "Finite element model" gives more questions than answers:

"what is depth of each individual ply"? Is it the size y-direction or z-direction? The size 0.132 is like a thickness of a ply. Only one element in thickness direction? Details about the number of elements/nodes has to be given.

Answer: The depth of each individual ply is 0.132 mm along the z direction, axes are defined in the revised manuscript and a typical FE mesh is provided in the new figure 5.

Is it a 3-D analysis as stated in the first sentence or 2D? There is nothing about edge effects (possible initiation at edges and propagation along fibers). Therefore I conclude that the analysis was 2D.

Answer: It is a 3D model. A figure to illustrate the 3D model with dimensions and boundary conditions has been added in the manuscript, see Fig. 5.

Was the whole specimen modeled or repeating elements of certain length (density) considered

Answer: The size of the model used is 10mm x 10mm to represent the area of cracks generated based on a certain crack density which is needed to simplify the model and reduce the computing time. It could be viewed as an RVE approach that uses repeating elements of certain length.

How about the effective constraint? Was it used or each layer was modeled separately? If so, boundary conditions have to be described that give "repeating element"

Answer: In this FE model, a displacement was applied at both ends of the plate, as shown in Fig. 5. The applied displacement is calculated based on the material properties and the stress value measured by the experiment. The corresponding description was added in the first paragraph of section 3.2 in the manuscript.

"all the 90-plies were located in the middle plane of the laminate" is an incorrect expression

Answer: The manuscript has been changed.

"the stiffness will be gradually degraded" is the stiffness of the cohesive element not the material

Answer: Text has been corrected.

"and a crack density of 2 cracks/mm was assumed…… which corresponds …. to 20 cracks per cm" is really a very deep and correct explanation. Should it be given?"

Answer: Text has been modified

9. "Results and discussion and conclusions

a. "the fracture model was found to depend on this ratio, so the same fracture parameters were used for both lay-ups" What does it mean?

Answer: Based on the experimental measurement, the G_{IC}, G_0 and R will influence the predicted accuracy using ECM for different thickness ratios. For the prediction of $[0/90]_s$ and $[25/-25/90_2]_s$ the stacking sequence and thickness of laminates are different but the thickness ratio is same (=1). So the same parameters of G_{IC}, G_0 and R were used for ECM prediction of these two lay-ups, see also previous comments.

b. "mesh refinement can slightly improve the accuracy…". This is NOT what we see in Fig. 7. We see that refinement is REDUCING THE AGREEMENT with test data at low crack density,

Answer: For the $[0/90]_s$ lay-up, the initial crack was found at a little higher stress value when the refined model was used but the initial crack density value was reduced to 1 crack/cm which is well matched with experimental data than that predicted (2 cracks/cm) by the relatively coarse model. Moreover, it did also improve the crack density for the $[25/-25/90_2]_s$. But, improvements are relatively small and this is why the coarser mesh is recommended that speeds up the solution and results are acceptable taking into account the experimental uncertainties in measuring fracture parameters or accurately measuring crack densities.

c. It is pointless to discuss 0.5mm or smaller distance between cohesive elements. The distance is scaled with the size of the crack (90-layer thickness) and it has to be discussed in these terms. By the way, the ply thickness should be given in Table 1. "

Answer: For the FE method, due to the initial size of model (distance defined between cohesive elements) was determined based on the experimental observation, it needs to be investigated the mesh size effect for the prediction for a different crack density was defined at a saturation level. In addition, the mesh dependency of cohesive elements were unknown for this simulation, it is more important to perform a refined model with refined size of the whole model (including cohesive elements). The results showed the refinement did not give much improvement but the crack density was accurately predicted in the same stress range when compared to the experimental data; this gives confidence to the proposed FE method, which is a reliable way to predict crack density and identify parameters that have an effect when simulating fracture of complex laminated structures. It is also a way of validating failure criteria, stress and/or fracture based.

The ply thickness has been added in Table 1.